A new paradigm for the universe

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A new paradigm for the universe is presented in which the universe is far older than current estimates and there is no big bang.

83F05; 83C15, 83C40, 83C57

Preamble

The purpose of this monograph is to present a new paradigm for the universe in which there is no big bang and in which the age of the universe is much larger than current estimates. Strictly speaking, it is not possible to define the age of the universe in this paradigm, because it does not have a universal time parameter and elapsed time depends on the world line chosen. This is of course a familiar property of general relativity, and it is worth remarking at once that this work does not propose any new physics and the whole program can be seen as strong supporting evidence for the correctness of standard Einsteinian relativity.

The work is incomplete. We do not yet have the correct metric for a rotating black hole. We know that the Kerr metric is not correct (because it is not Machian) and we can give many properties of the correct metric. Nor do we have a full description of a suitable space-time for the universe, though we have some ideas and properties. (More details on these, and on every other topic mentioned in the overview which follows, will be given later, see the contents below.) Nevertheless we think that it is useful to present this preliminary, fargmentary version because many of the puzzling aspects of observations which are not fully explained by current cosmology are already given satisfactory explanations and it is possible to gain a good idea of the final shape of the full paradigm.

This work overturns nearly every tenet of current cosmology. It does not however contradict any observational evidence. Explanations are given for a good number of observations that are currently interpreted as contradicting our hypotheses, but undoubtedly there are many that are not yet included. Please read carefully to the end before coming to a conclusion about the work. It is obviously not possible to address all burning questions immediately but we hope that they are adressed somewhere.

Large parts of this work are joint work with Robert MacKay.

1 Overview

We have made an effort to make this overview accessible to the widest possible readership. Technical details will be given in later sections for every point made here and we ask experts to be patient.

The main hypothesis is that an ordinary spiral galaxy (such as the milky way) contains a hypermassive central object, commonly called a black hole, of mass up to 10^{14} solar masses. The terminology “black hole” is very unfortunate as it is already clear that hypermassive objects are anything but black; however it is well established and we shall continue to use it. This central mass controls the dynamics of the galaxy and is responsible for generating and maintaining the predominant spiral structure. Presently we use a combination of relativistic and Newtonian arguments to explain this dynamic. A fully relativistic explanation is needed.

One of the strongest pieces of direct evidence for a hypermassive central mass in a normal galaxy is now nearly 80 years old. In 1933 Zwicky [14] used the virial theorem to estimate the mass of galaxies in the Coma Berenices cluster and discovered that the mass exceeds luminosity mass by a factor of 10^2 to 10^3. His estimate of the average mass of a galaxy in the cluster was 10^{11} to 10^{12} solar masses. Since the cluster contains many objects which are not full size galaxies, this puts the mass of a full size galaxy at around 10^{12} to 10^{13} solar masses. See the comments below on the nature of galaxy/quasar objects and their mass ranges. More recent observations have used the same methods to find masses up to 10^{14}.

In current cosmology the missing mass is labelled “dark matter”, ie matter for which there is no direct evidence, and is arranged in a rough sphere with the bulk well outside the luminous disc. The outer dark matter is used to explain observed rotation curves, on the assumptions that there are no significant relativistic effects and that stars all move on roughly circular orbits.

Observed rotation curves for galaxies are quite striking. Essentially the curve (of tangential velocity against distance from the centre) comprises two approximately straight lines with a short transition region. The first line passes through the origin, in other words rotation near the centre has constant angular velocity (plate-like rotation) and the second is horizontal; in other words the tangential velocity is asymptotically constant, Figure 1 (left). Furthermore observations show that the horizontal straight line section of the rotation curve extends far outside the limits of the main visible part of galaxies and the actual velocity is constant within less than an order of magnitude over all galaxies observed (typically between 100 and 300km/s).

A short aside here. There are more recent observations of rotation curves using other wavelengths which show different behaviour near the origin, some fitting with a large central mass. All agree on the characteristic horizontal straight line. For a survey, see [13].

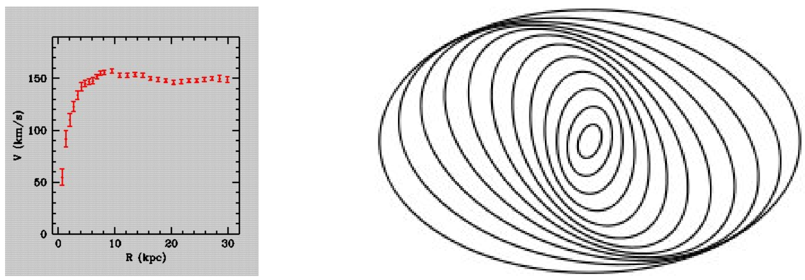

Figure 1: Left: the rotation curve for the galaxy NGC3198 reproduced from [1] (sourced from Begeman [6]). Right: the standing wave theory, reproduced from wikipedia

Galactic rotation curves are so characteristic (and simple to describe) that there must be some strong structural reason for them. They are very far indeed from the curve you would get with a standard Keplerian model of rotation under any reasonable mass distribution. Nevertheless conventional cosmology hypothesises that the dark matter mentioned above is distributed in precisely the right way to make Keplerian rotation fit the rotation curve. This is extremely implausible for several reasons. Firstly the quantity of dark matter is huge and tends to infinity with the radius of fit, which as mentioned above appears to be unbounded. Secondly the dark matter has enormous angular momentum. Since galaxies are supposed to have condensed out of primeval clouds of matter, the condensation must have happened in such a way as to cause these huge pools of angular momentum. This is a highly unlikely scenario and the problem of how it happened is known as the "angular momentum problem". There are some attempts to explain it, but none are compelling. Thirdly it is unreasonable to suppose that exactly the right distribution of dark matter happened (again by condensation) for every galaxy and fourthly, the final arrangement with most of the matter on the outside is dynamically unstable. For stability in a rotating system (such as the solar system or Saturn's discs) you must have a strong central mass to hold it together. Failing this the system will tend to condense into smaller systems. [This needs underpinning]

In our paradigm the missing matter is at the centre and there are no angular momentum or stability problems. Condensation which results in concentration of matter at centres is plausible and indeed similar to the arrangement for solar systems where the bulk of the mass is in the central star and the planets carry just a small residue of angular momentum. Moreover the correct form of the rotation curve arises naturally from interial drag effects (Mach's principle) due to the rotating central mass. The curve that we derive has a horizontal straight assymptote for large r which fits the observation of constant tangential velocity for unbounded r. By varying initial conditions we can fit the other curves observed.

It is worth remarking in passing that a heavy centre is a very natural hypothesis: for example the sun contains most of the mass in the solar system and the nucleus of an atom contains most of the mass. Thus, on general principles, you would expect that the centre of such a well organised structure as a spiral galaxy would account for all but a fraction of a percent of the mass.

Figure 2: M101 (left) and NGC1300 (right): images from the Hubble site [2]

There are other serious problems with the conventional dynamic arrangement for galaxies. Most normal galaxies have a clear and strikingly beautiful spiral structure (Figures 2 and 3). In order for a spiral structure to be maintained stably over several revolutions, with all stars moving on circular orbits, you need tangential velocity roughly proportional to distance from the centre which means that the rotation curve is far from the one observed, Figure 1 (left). To square this second circle, conventional cosmology proposes that the spiral arms are not real but virtual. It proposes that they are in fact "standing waves", Figure 1 (right). No comment is needed when comparing this with Figure 2 (right) or Figure 3 (left).[1]

In our paradigm the centre of a normal galaxy generates the spiral arms by a process whereby matter is ejected from a central belt structure and condenses into solar systems.

[1]To be a little fairer to the standing wave theory, there is a rider to the theory which suggests that a shock-wave effect causes short-life stars to appear as the standing wave moves. This rider would carry more force if there was any evidence for such short-life stars in our immediate neighbourhood: after all the sun is part of a spiral arm of the Milky Way.

 

Figure 3: NGC1365 and M51 images from NASA and Hubble site resp

Thus the familiar spiral structure is like the visible spiral structure in a Catherine wheel, the arms being maintained by stars moving along them, and there is no need for any special pleading to explain the observed structure. It follows that, in our paradigm, young stars in a galaxy are moving outwards as well as around the centre. This general outward movement has not been observed and the reason for this is that (for an observer on the same side of the centre) the frequency shift due to the outward motion is cancelled by the gravitational frequency shift from the gravitational field of the centre. Stars move outwards at near escape velocity, so the two opposing frequency shifts are almost the same. Also the motion of stars is far from Keplerian, being strongly controlled by inertial drag effects from the (rotating) centre. The result is that the outward progress takes a very long time—commensurate with the lifetime of a star and hence the outward velocity is rather smaller than (about one tenth of) the observed rotational velocity. The general picture which emerges is of a structure stable over an extremely long timescale (at least 10^{12} years) with stars born and aging on their outward journey from the centre and returning to the centre to be recycled with new matter to form new solar systems. Our tentative suggestion is that galaxies have a natural lifetime of perhaps 10^{16} years with the universe considerably older than this.

The generator for spiral arms is analogous to the inner belt of the accretion disc hypothesised for active galactic nuclei and used to explain their observed radiation. Indeed one very helpful way to understand our paradigm is that it fits ordinary galaxies into a spectrum of black hole based phenomena starting with quasars proceeding through active galaxies and on to to full size spirals, with the mass of the centre increasing monotonically along the spectrum. All are highly active. A tentative guess at the mass ranges of the central black hole (in solar masses) along the spectrum is:

Quasars: 10^7 to 10^9

"Active" galaxies: 10^9 to 10^{12}

"Normal" galaxies: 10^{12} to 10^{14}

Proceeding along the spectrum, the mass of accreted matter increases and progressively hides the centre, so that received radiation comes from matter less and less directly affected by the central black hole. This is the reason why the active nature of normal galaxies has not been directly observed—their centres are totally shielded from view.

A short aside here: there is a small industry working on the central mass of active galaxies using "virial methods". The virial theorem requires an isolated system and does not apply to a partial system such as the bulge in the centre of a galaxy. Nevertheless it is possible to make plausible estimates for central masses, assuming that the virial theorem applies locally, which are found to be typically in the range 10^7 to 10^9. The methods assume that the stars near the centre can be observed and hence that the mass of the centre will be corellated with observed velocity dispersion for these stars. This assumption breaks down for normal galaxies, where, as we have seen, the central mass is densely shielded from outside view, so our hypothesis of a hypermassive centre for a normal galaxy is consistent with this body of observations and interpretation.

As remarked above, observed radiation from near the massive centre in fact comes from matter progressively further out as the central mass increases along the spectrum. A massive centre causes gravitational redshift. We would therefore expect that there should be instrinsic redshift (ie redshift not due to cosmological effects) observed at the lower end of the spectrum, ie that quasars should typically exhibit instrisic redshift. A recent paper of Hawkins [8], which sets out to prove that quasars show redshift without time dilation (an impossibility since redshift and time dilation are identical), in fact decisively proves that much of the redshift observed in quasars is intrinsic. This provides direct evidence for our proposed black hole spectrum. The idea that quasars typically exhibit instrinsic redshift has a strong champion in Halton Arp [5]. Apparently this view is controversial. As a newcomer to this subject, I find this controversy bizzarre: it is very natural to expect that there will be heavy objects in the universe and that nearby sources of radiation should be redshifted by gravitational redshift independently of cosmological effects.

We need at this point to briefly mention two other areas of observation. Firstly there is another small industry monitoring orbits of stars around a small black hole (of mass about 10^6 solar masses) at Sagittarius A^* which is erronously held to be the centre of our galaxy. As we have seen, the centre of our galaxy is deeply hidden. Moreover there is clear evidence (from the COBE satellite) that Sagittarius A^* is not located at the centre. Secondly there has been a huge effort expended mapping the velocities of stars in our neighbourhood. There are some paradoxical properties of these excellent observations. In particular, the symmetries in velocity variations that you would expect from the current dynamical model of the galaxy (with stars moving in circular orbits) are not observed. The "velocity ellipsoid" which expresses this variation does not have the line from the sun to the galactic centre as a principal axis, as would be expected from symmetry; the deviation of these two directions is called "vertex deviation". Further, vertex deviation varies systematically with stellar age. With our dynamical model, the paradoxical aspects disappear and vertex deviation and its corellation with age, have very natural explanations. This, like everything else in this overview, will be discussed in greater detail later, as will the real nature of the Sagittarius cluster.

Before moving on to global cosmological considerations it is worth briefly commenting that there is a natural way to see galaxy/quasar objects as evolving over an extremely long timescale with points of the spectrum representing different ages of the same class of objects. Moreover there is also a natural way for the rate of rotation to stabilise which explains why the observed limiting tangential velocity is more-or-less the same for different galaxies. It is worth remarking that these ideas fit in with the observations of Arp and others [5], which show quasars closely connected with parent galaxies and (intrinsic) redshift decreasing with age. However we not support the more outlandish theories that these authors suggest as a theoretical framework. As far as we are concerned, these observations all have natural explanations within standard relativity, for example the decreasing intrinsic redshift with age is a natural consequence of the masking effect of the centre described above.

Thus far we have ignored the wider context of cosmology. Our hypotheses for galaxies are consistent with a lifetime of perhaps 10^{16} years which is several orders of magnitude greater than the current estimate of the age of the universe of about 10^{10} years. Thus the current theory with a hot big bang about 10^{10} years ago must be wrong. There are three pieces of primary evidence for the big bang: the distribution of light elements, the cosmic microwave background, and redshift. We propose new explanations for all three. The distribution of light elements is the easiest. Visible matter in the universe is mostly associated with galaxies and hence has arisen in the way that we described above for solar systems and spiral arms, namely by being ejected from the generating belt near the centre. This belt has a complex layered structure (similar, as mentioned above, to the accretion belt around an active galaxy) with an inner layer of pure energy surrounded by layers where the energy progressively condenses into plasma and then into ordinary matter. The conditions are similar to those hypothesised near the big bang and the resulting mix of light elements is therefore the same. To explain the two other pieces of primary evidence requires a careful investigation into the nature of space-time. Our point of view (in common with relativity and quantum theory) is that all phenomena must be related to observers. We call an observer moving along a geodesic a "natural" observer and our basic idea is to consider natural observer fields, ie a continuous choice in some region of natural observers who agree on a split of space-time into one time coordinate and three normal space coordinates. Within a natural observer field there is a coherent sense of time: a time coordinate that is constant on space slices and whose difference between two slices is the proper time measured by any observer in the field.

We consider that the high-z supernova observations [3, 4] decisively prove that redshift (and consequent time dilation) is a real phenomenon. In a natural observer field, red or blueshift can be measured locally and corresponds precisely to expansion or contraction of space measured in the direction of the null geodesic being considered. Therefore, if you assume the existence of a global natural observer field, an assumption made implicitly in current cosmology, then redshift leads directly to global expansion and the big bang. But there is no reason to assume any such thing and many good reasons not to do so. It is commonplace observation that the universe is filled with heavy bodies (galaxies) and it is now widely believed, independently of our main hypothesis, that the centres of many galaxies harbour massive black holes. The neighbourhood of a black hole is not covered by a natural observer field. You do not need to assume that there is a singularity at the centre to prove this. The fact that a natural observer field admits a coherent time contradicts well known behaviour of space-time near an event horizon.

We sketch the construction of a universe in which there are many heavy objects and such that, outside a neighbourhood of these objects, space-time admits natural observer fields which are roughly expansive. This means that redshift builds up along null geodesics to fit Hubble's law. However there is no global observer field or coherent time or big bang. The expansive fields are all balanced by dual contractive fields and there is in no sense a global expansion. Indeed, as far as this makes sense, our model is roughly homogeneous in both space and time (space-time changes dramatically near a heavy body, but at similar distances from these bodies space-time is much the same everywhere). A good analogy of the difference between our model and the conventional one is given by imagining an observer of the surface of the earth on a hill. He sees what appears to be a flat surface bounded by a horizon. His flat map is like one natural observer field bounded by a cosmological horizon. If our hill dweller had no knowledge of the earth outside what he can see, he might decide that the earth originates at his horizon and this belief would be corroborated by the strange curvature effects that he observes in objects coming over his horizon. This belief is analogous to the belief in a big bang at the limit of our visible universe. This analogy makes it clear that our model is very much bigger (and longer lived) than the conventional model. Indeed it could be indefinitely longlived and of infinite size. However there is evidence that the universe is bounded, at least as far as boundedness makes sense within a space-time without universal space slices or coherent time. Our construction is based on de Sitter space, and hence there is a nonzero cosmological constant, but this is for convenience of exposition and we believe that a similar explanation for redshift works without a cosmological constant.

To explain the cosmological microwave background (CMB) we recall the Hawking effect which applies at any horizon [7]. It needs stressing that horizons are observer dependent. The definition of the horizon for an observer in a space-time is the boundary of the observer's past and represents the limit of the observable universe for her/him. The event horizon for a black hole is part of the horizon for any observer outside (but NOT for an observer on or inside this surface). The fact that many observers share this part of their horizons leads to a common mistaken belief that this surface is in some way locally special. It is not. For an observer on or inside this event horizon it has no special significance. The best way to think of a horizon is as a mirage seen by the observer, not a real object. The key point about a horizon is that it shields the observer from knowledge of the other side. Ignorance is entropy in a very precise technical sense. And if you have perfect entropy in a system then the radiation that comes off is black body (BB).

Gibbons and Hawking [7] apply these idea to de Sitter space where there is a cosmological horizon for any natural observer. Moreover the symmetry of de Sitter space gives perfect isotropy which is one of the salient features of CMB. The radiation that they use comes from quantum fluctuations in space-time (particle pairs are produced and one half passes over the horizon whilst the other stays to be observed). This produces BB radiation at $10^{-28}\,{}^\circ$K which is far lower than observed. We apply the same ideas (again in de Sitter space) to the background of photons coming from all the galaxies in the universe and we use the fact that the universe is also filled with low-level gravitational waves as seen in the systematic distortion in distant images from the Hubble telescope. These two considerations have the effect of greatly accelerating the Hawking effect and bring the temperature up to the 5°K observed. Looking at the cosmological horizon we see light particles at near zero energy because they have traversed the Hubble radius. The low-level gravitational waves cause these particles to appear to be moving back and forth across the boundary. Think of the waves as like parcels of space-time being swept over the boundary in both directions. Particles get swept away and lost (from view) and appear to be being absorbed, and others get swept over the boundary towards us and appear to be being created. This is an exact paradigm for absorption and re-emission.

So the particles that are emitted (ie what we see) give a black body spectrum.

It is also worth commenting that the same circle of ideas gives a natural and non-catasclysmic explanation for gamma ray bursts (GRB). Black holes in de Sitter space follow geodesics (you can think of the metric as roughly given by plumbing in a Schwarzschild black hole along a geodesic). If you consider the light paths from one generic geodesic to another in de Sitter space you find that there is a first time when they communicate (when the emitter passes through the horizon of the receiver) and a last time (when the dual occurrence happens and the receiver passes through the boundary of the future of the emitter). It turns out that the apparent velocity is infinite (infinite blueshift) when they first communicate and zero when they last communicate (infinite redshift). We suggest that the universe is closely enough modelled by de Sitter space that distant galaxies coming across our horizon at "infinite blueshift" but at Hubble distance cause the observed GRB.

2 Contents

This monograph is still very much "under construction" and in the meantime the content is to be found in other papers which are linked below.

PART I : the geometry of space-time

All of this part is joint work with Robert MacKay.

1 Basic definitions and notes on de Sitter space

We use de Sitter space as a convenient framework for our model for space-time and this section provides a convenient place to collect known facts that we will use.

The current version of this section is to be found in "Notes on de Sitter space":
http://msp.warwick.ac.uk/~cpr/paradigm/notes.pdf

2 Natural flat observer fields in spherically-symmetric space-times

This section, which is based on a joint paper [9], proves that spherically-symmetric space-times such as Schwarzschild and Schwarzschild–de-Sitter space admit two natural observer fields with flat space slices. One is the escape field and the other the dual capture field.

For the current version see:
http://msp.warwick.ac.uk/~cpr/paradigm/escape-submit-ref.pdf

3 Natural observer fields and redshift

Based on another joint paper [10], this section contains a sketch of the proposed geometry for the universe. The natural observer fields found for Schwarzschild–de-Sitter space are patched together to construct a universe in which there are many heavy objects and such that, outside a neighbourhood of these objects, space-time admits natural observer fields which are roughly expansive. This means that redshift builds up along null geodesics to fit Hubble's law. However there is no global observer field or coherent time or big bang.

For the current version see:
http://msp.warwick.ac.uk/~cpr/paradigm/redshift-nat-final.pdf

4 Gamma ray bursts

Again based on a joint paper [11], this section contains full details of the alternative explanation for GRB outlined above.

For the current version see:
http://msp.warwick.ac.uk/~cpr/paradigm/GammaRayBursts.pdf

5 The cosmic microwave background

This section (in preparation) contains full details of the alternative explanation for CMB outlined above.

PART II : the structure of galaxies

1 A new paradigm for the structure of galaxies

This section contains details of the derivation of the dynamic of a galaxy from Mach's principle. The engine is inertial drag from the massive rotating central mass. We find a full set of equations for the dynamic using Newtonian physics plus inertial drag. We believe that inertial drag is the main relativistic effect in this system and that this model gives a good approximation to a fully relativistic model. We use Mathematica to sketch the shapes of galaxies under various initial conditions and find sketches that fit well with observed galaxies. A preliminary sample is given in Figure 4. On the left are graphs of the inertial tangential velocity of a star against radial distance r (blue graph), the velocity required to overcome gravity (red graph) and the difference (bottom). Because the graphs are similar, we make the simplifying assumption that outward velocity is constant. The resulting spiral structure is shown on the right. In the next version of this section we will model galaxies more accurately by omitting this simplifying assumption. Units are years throughout ($c = 1$, time measured in years and distances in light years). It can be checked that the numbers are a reasonable fit for a full-size spiral such as M101 or M51.

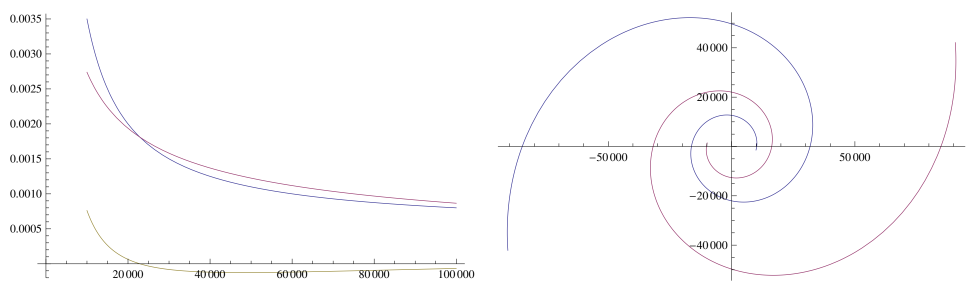

Figure 4: Mathematica models for galactic dynamics

In this section also is a discussion of the origin and prevalence of solar systems and a tentative explanation for the rapid origin of life.

For the current version see:
http://msp.warwick.ac.uk/~cpr/paradigm/struc-gal.pdf

2 Notes on rotation metrics

This section (in preparation) proves that the the Kerr metric does not give the correct rotation curve and hence is not Machian. Properties of the correct metric for a rotating black hole are discussed.

3 Stability of rotating systems

(In preparation) It is proved that the conventional distribution of matter in a galaxy with most of the matter at the outside is dynamically unstable.

PART III : observations

1 Optical distortion in the Hubble Ultra Deep Field

http://msp.warwick.ac.uk/~cpr/paradigm/HUDF-2.pdf

There is clear evidence of optical distortion caused by gravitational waves in the Hubble Ultra Deep Field.

2 Interpreting observed local steller velocities

http://msp.warwick.ac.uk/~cpr/paradigm/velocity.pdf

The remarkable features of observed local stellar velocities are fully explained within the new paradigm.

3 A note on Sagittarius A^*

http://msp.warwick.ac.uk/~cpr/paradigm/sgrastar.pdf

Sgr A^*, together with the stars which orbit it, form an old, nearly totally collapsed, globular cluster, which it is not at the centre of the galaxy and there is no reason to suppose that it is at rest.

4 Intrinsic redshift in quasars

http://msp.warwick.ac.uk/~cpr/paradigm/hawkins-time-dilation.pdf

A recent paper by M R S Hawkins "On time dilation in quasar light curves" [8] conclusively proves that quasars have instrinsic redshift.

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